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Some Results For M/C2(a, D, B)/1 Queueing System

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Abstract:

In this paper a bulk-service queuing system is being considered which have a single server and arrivals occur singly according to a Poisson process. Service times have a pdf of Coxian type of order 2. Service is provided in batches where a batch may contain a minimum number of $a \geq 1$ and a maximum number of $b \geq a$ customer. It is further assumed that the arriving units can enter service, without affecting the service time, if the size of the batch served is less than some fixed integer d ($a \leq d \leq b$). The steady state distribution of queue length and waiting timeare derived along with probability that the server is busy and the customer has to wait. These types of queues are widely found in transportation and production networks.

Key Words: service, Accessible batch, Non-accessible batch, single server, Coxian distribution.

In transportation networks, it usually happens that customers are served in bulk and late entries are allowed to join a batch, if the capacity of the batch is not full. Customers may be persons, luggage, vehicles, data, raw material or anything requiring service. Single-server bulk-service queuing systems with either the (a, b) rule and its special cases or with other service rules have got a wide attention. Most of the early research was restricted to exponential service times. Chaudhry and Templeton (1981), Holman et.al.(1981) and Baba (1983) studied the M/ G(a, b)/1 queuing system and some of its special cases like the M/ E_k (a, b)/1,M/G(1, b)/1 and M/PH(a,b)/1. Jacob and Madhusoodanan (1988) gave transient solution to M/ G(a, b)/1 queuing system.

Bulk service queues with accessible and non-accessible batches are studied by Chiamsiri and Leonard (1981), and Sivaswamy (1990). Goswami et.al. (2011) didperformance analysis of a renewal input bulk service queue with accessible and non-accessible batches. Balasubramanian (2013)gave steady state analysis of a bulk queuing model with multiple vacations, accessible batches and closedown times. Banerjee et.al. (2015) analyzeda finite-buffer bulk service queue underMarkovian arrival process with batch- size dependent service. Here, an attempt has been made to solve a single server queuing system with service in accessible and non-accessible batches having Poisson arrivals and service time pdf of Coxian-2 type.

MODEL AND NOTATIONS

We assume that customers arrive into a service facility according to a Poisson process with rate λ. Service is provided in groups by a single server. When the server becomes free, a group of customers of size at most b can be served. The server is not allowed to process a group of size less than a, $1 \leq a \leq b$. It has further been assumed that late entries can join a batch in course of ongoing service as long as the number of customers in that batch is less than d (called maximum accessible limit) where $a \leq d \leq b$. If the batch size is greater than or equal to d, it becomes non-accessible for late arriving customers. If the server is accessible, then late arriving customers will join the batch being processed till it becomes non-accessible and only after that the queue will be formed. The late entries into a batch does not affect the service time of the batch being served.

The service times, irrespective of the batch size are assumed to be independent and identically distributed random variables with a common Coxian-2 type distribution C_2. The queue discipline is FCFS.

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The Coxian distribution with two stages can be characterized as follows:

The service facility consists of 2 exponential stages with corresponding rates μ_1 and μ_2, only one of which may be occupied at any moment. Customers enter to the first stage, but after visiting the first stage, an independent choice is made so that the customers traversing the facility either departs with probability q or proceeds to the next stage with probability 1-q . If he visits the second stage, then after it, he has to leave the service facility with probability one. So, the C_2 distribution has three independent parameters, i.e.

μ_i ≡ the rate of the i^{th} exponential stage, i=1,2

q≡the probability of bypassing the second stage

Let

T_s≡ the customer's total service time,

then, its squared coefficient of variation $v_s{}^2$ and Laplace transform $f_{T_s}{}^*(\theta)$ are found to be respectively

$$v_s{}^2 = \frac{\mu_1{}^2\,(1-q^2)+\mu_2{}^2}{[\mu_1(1-q)+\mu_2]^2} \text{ and } f_{T_s}{}^*(\theta) = \frac{\mu_1\mu_2+\mu_1\, q\, \theta}{(\theta+\mu_1)\,(\theta+\mu_2)}.$$

For the above discussed queueing model M/ C_2 (a, d, b)/1, expressions are found for the steady state probability distribution of queue length along with some usual performance measures.

For the steady state, we introduce the following random variables

(R, 0) ≡ R server busy with non-accessible batch,R=0,1

(R, 1) ≡ R server busy with accessible batch,R=0, 1

N≡ The number of customers in the queue,

N_0 ≡ The number of customers being served in an accessible batch,

B≡ The number of busy server in the second stage of its service-timing-channel.

The system is studied as a continuous time Markov chain, by using the triplets ((r, 0), n,i) to represent the states ((R,0) = (r,0), N=n, B=i) and ((r,1), n_0, i) to represent the states ((R,1) = (r,1), N_0 = n_0, B=i) on the three-dimensional state space

$$S = \{((r, 0) = (0, 0)) \times ((0 \le n \le a-1) \times (i = 0)\} \cup \{((r,1) = (1,1) \times (a \le n_0 \le d-1) \times (i= 0,1)\}$$

$$\cup \{((r,0) = (1,0)) \times (n \ge 0) \times (i=0,1)\}$$

where the states with (r,0) = (0,0) and (r,1) = (1,1) are termed 'unsaturated' and the infinitely many states with (r,0) = (1,0) are termed as 'saturated'.

Let us define the general time-probabilities as

$P_{(r,o),n,i}$ ≡ Pr {(R,0) =(r,0), N =n, B=i },((r,0),n,i)∈ S

$P_{(r,1),n_o,i}$ ≡ Pr {(R,1) =(r,1), N_0 =n_0, B=i },((r,1),n_0,i)∈ S

P_n ≡ Pr{ N=n}

U_n = Pr {N=n : the system is unsaturated}

So,

$U_n = P_{(0,0),n,0}$ for n ≥ 1

and $U_0 = P_{(0,0),0,0} + \sum_{n_0=a}^{d-1} \sum_{i=0}^{1} P_{(1,1),n_0,1}$

where for compactness, we define

$P_{(r,0),n,r+1} \equiv P_{(r,0),n,-1} \equiv 0$

$P_{(r,1),n_0,r+1} \equiv P_{(r,1),n_0,-1} \equiv 0$

As well as the traffic intensity$\rho \equiv (\lambda/b)[(1/\mu_1) + (1-q)/\mu_2]$

THE EQUILIBRIUM EQUATIONS

Applying the well-established conservation of flow argument, equilibrium equations can be written as:

1. For (r,0) = (1,0)

(a) $n \geq 1$, i=0,1

$$\lambda P_{(1,0),n-1,i} + \mu_1(1-q)(2-i)P_{(1,0),n,i-1} + q\mu_1(1-i)P_{(1,0),n+b,i} + \mu_2(i+1)P_{(1,0),n+b,i}$$

$$=(\lambda + (1-i)\mu_1 + i\mu_2)P_{(1,0),n,i} \quad (1)$$

(b) n=0, i=0,1

$$\lambda P_{(1,1),d-1,i} + (1-i)\mu_1 q \sum_{k=d}^{b} P_{(1,0),k,i} + (i+1)\mu_2 \sum_{k=d}^{b} P_{(1,0),k,i+1} + \mu_1(2-i)(1-q)P_{(1,0),0,i-1}$$

$$= (\lambda + (1-i)\mu_1 + i\mu_2) P_{(1,0),0,i} \quad (2)$$

2. For (r,1) = (1,1)

(a) $a+1 \leq n_0 \leq d-1$, i= 0, 1

$$\lambda P_{(1,1),(n-1)_0,i} + \mu_1(1-q)(2-i)P_{(1,1),n_0,i-1} + \mu_1 q(1-i)P_{(1,0),n,i} + (i+1)\mu_2 P_{(1,0),n,i+1}$$

$$= (\lambda + (1-i)\mu_1 + i\mu_2)P_{(1,1),n_0,i} \quad (3)$$

(b) $n_0 = a$, i= 0,1

$$\lambda P_{(0,0),a-1,i} + q\mu_1(1-i)P_{(1,0),a,i} + (i+1)\mu_2 P_{(1,0),a,i+1} + \mu_1(1-q)(2-i)P_{(1,1),a,i-1}$$

$$= (\lambda + (1-i)\mu_1 + i\mu_2) P_{(1,1),a,i} \quad (4)$$

3. For (r,0) = (0,0)

(a) $1 \leq n \leq a-1$, i=0

$$\lambda P_{(0,0),n-1,0} + q\mu_1 P_{(1,0),n,0} + \mu_2 P_{(1,0),n,1} = \lambda P_{(0,0),n,0} \quad (5)$$

(b) n=0, i=0

$$q\mu_1(P_{(1,0),0,0} + \sum_{n_0=a}^{d-1} P_{(1,1),n_0,0}) + \mu_2(P_{(1,0),0,1} + \sum_{n_0=a}^{d-1} P_{(1,1),n_0,1}) = \lambda P_{(0,0),0,0} \quad (6)$$

3. Analysis and solution of the equilibrium equations

We assume that

$P_{(1,0),n,i} = B_i w^n$, $n \geq 0$, i=0,1

From equations (1), we have

$$B_{i-1}\, w(1-q)(2-i)\mu_1 \; + \; B_i[\lambda + q(1-i)\mu_1 w^{b+1} \; - \; w(\lambda \; + \; (1-i)\mu_1 \; + i\mu_2)]$$

$$+B_{i+1}(i+1)\mu_2 w^{b+1} \; = \; 0$$

i = 0,1 (7)

Equations (7) form a linear homogeneous system of two equations with two unknowns. The non-trivial solution will exist only if the determinant of this system matrix equals to zero, i.e.

Det(w) = 0 (8)

Using Rouche's theorem in above equation, it is easy to see that for $\rho < 1$, there are two real and positive roots w_j , j =0,1inside the unit circle.

Let us introduce the generating function $U(z) = \sum_{i=0}^{1} B_i z^i$

Now, after multiplying (7) by z^i and adding with respect to I, we obtain the separable differential equation of first order

$\dot{U}(z)/U(z) = A(w) / [z - z_1(w)] - [1 - A(w)] / [z - z_2(w)]$

where $z_1(w)$ and $z_2(w)$ are the roots of the denominator. It gives

$U(z) = k\,[z - z_1(w)]^j\,[z - z_2(w)]^{1-j}$,j=0,1 (9)

where k is a constant yet to be determined.

Returning now to the assumed form of probabilities $P_{(1,0),n,i}$, we have

$P_{(1,0),n,i} = \sum_{j=0}^{1} B_{i,j}\, w_j{}^n$,n≥ 0, i= 0,1

with $B_{i,j}$being the coefficient corresponding to the root w_j .These coefficients $B_{i,j}$ satisfy equation (7) for the two roots w_j .

As the equations given by (7) are linear homogeneous, only one of them is independent. So, for each j the coefficient $B_{0,j}$ can be expressed as linear combinations of $B_{i,j}$. If we introduce the notations

$C_j{}^{*} \; \equiv \; B_{1,j}$ forj=0,1 (10)

$B_j{}^{*} \; \equiv \; C_j{}^{*}(1 - z_1(w_j))^j\,(1 - z_2(w_j))^{1-j}$ (11)

It is apparent that we must have

$B_{i,j} \; = \; f(i, w_j)\; C_j{}^{*}$i, j = 0,1

where the coefficients $f(i, w_j)$ can be determined recursively through equation (7). Hence ,

$P_{(1,0),n,i} = \sum_{j=0}^{1} f(i, w_j)\; C_j{}^{*}\, w_j{}^n$,n≥ 0, i= 0,1 (12)

Now, (9) and (10)lead to

$U_j(z) \; = \; \sum_{i=0}^{1} B_{i,j}\; z^i = C_j{}^{*}(z - z_1(w_j))^j\,(z - \; z_2(w_j))^{1-j}$,j=0,1

And therefore the coefficients $f(i, w_j)$ satisfy the two relations

$$\sum_{i=0}^{1} f(i, w_j)\, z^i = (z - z_1(w_j))^j (z - z_2(w_j))^{1-j}, j=0,1 \tag{14}$$

Now, the only unknown quantities are the two coefficients C_j^* and the probabilities

$P_{(0,0),n,0}$ with $0 \le n \le a-1$ and $P_{(1,1),n_0,i}$ with $a \le n_0 \le d-1, i=0,1$

From (2), we see that $P_{(1,1),d-1,i}$ can be expressed as linear combinations of the probabilities $P_{(1,0),n,i}$ and hence from (12) as linear combinations of C_j^*, i.e.

$P_{(1,1),d-1,i} = \sum_{j=0}^{1} C_j^* g(d-1, i, j)$, where the coefficients g(d-1, i, j) are to be computed recursively from (2).

Similarly, from (3), $P_{(1,1),(n-1)_0,i}$ are computed recursively for $a+1 \le n_0 \le d-1$. From (4), $P_{(0,0),a-1,i}$ is computed, while from (5) $P_{(0,0),n-1,0}$ is computed for $1 \le n \le a-1$. In all these cases, probabilities are to be expressed as linear combinations of C_j^*'s. So, we have

$$P_{(1,1),n_0,i} = \sum_{j=0}^{1} C_j^* g(n_0, i, j) \tag{15}$$

and $$P_{(0,0),n,i} = \sum_{j=0}^{1} C_j^* g(n, i, j) \tag{16}$$

The coefficients g(a-1,1,j) computed from (4) obviously satisfy

$$\sum_{j=0}^{1} C_j^* g(a-1, 1, j) = P_{(0,0),a-1,1} = 0 \tag{17}$$

Normalization equation gives

$$\sum_{n=0}^{a-1} P_{(0,0),n,0} + \sum_{n_0=a}^{d-1} \sum_{i=0}^{1} P_{(1,1),n_0,i} + \sum_{n=0}^{\infty} \sum_{i=0}^{1} P_{(1,0),n,i} = 1 \tag{18}$$

Equation (18) may be written as eq. (19) given below

$$\sum_{j=0}^{1} C_j^* \left[\sum_{n=0}^{a-1} g(n, 0j) + \sum_{n_0=a}^{d-1} \sum_{i=0}^{1} g(n_0, i, j) + \{1/(1-w_j)\} \left(1 - z_1(w_j)\right)^j \left(1 - z_2(w_j)\right)^{1-j} \right] = 1 \tag{19}$$

Equations (17) and (19) form a linear system of two equations with two unknowns C_0^* *and* C_1^*. *By solving them for* C_j^*, $j = 0,1$ *all probabilities are determined.*

System Size Probability Distributions and System Performance Measures

We have

$$P_n = \begin{cases} \sum_{i=0}^{1} P_{(1,0),n,i} \text{ for } n \ge a \\ P_{(0,0),n,0} + \sum_{i=0}^{1} P_{(1,0),n,i} \text{ for } 1 \le n \le a-1 \\ P_{(0,0),0,0} + \sum_{i=0}^{1} P_{(1,0),0,i} + \sum_{n_0=a}^{d-1} \sum_{i=0}^{1} P_{(1,1),n_0,i} \text{ for } n = 0 \end{cases}$$

From (12) and (14), we get

$$\sum_{i=0}^{1} P_{(1,0),n,i} = \sum_{j=0}^{1} (1 - z_1(w_j))^j (1 - z_2(w_j))^{1-j} C_j^* w_j^{\,n} = \sum_{j=0}^{1} B_j^* w_j^{\,n} \tag{20}$$

The definition of U_n with (20) gives

$$\begin{cases} \sum_{j=0}^{1} B_j^* w_j^n \; for\; n \geq a \\ U_n + \sum_{j=0}^{1} B_j^* w_j^n \; for\; 1 \leq n \leq a-1 \\ U_0 + \sum_{j=0}^{1} B_j^* \; for\; n = 0 \end{cases} \quad (21)$$

Now, we have to find the closed form expressions for U_n . From equation (5), we have

$$U_n - U_{n-1} = (1/\lambda)\,[q\mu_1 \sum_{i=0}^{1}(1-i)\, P_{(1,0),n,i} + \mu_2 \sum_{i=0}^{1} i\, P_{(1,0),n,i}]$$

for n = 1,2,…,a-1 (22)

After some tedious algebra, we obtain from equation (1)

$$q\mu_1 \sum_{i=0}^{1}(1-i)\, P_{(1,0),n,i} + \mu_2 \sum_{i=0}^{1} i\, P_{(1,0),n,i} = \lambda \sum_{j=0}^{1} B_j^* \,[(1-w_j)/1-w_j^{\,b}]w_j^{\,n-1} \quad (23)$$

So, from (22) and (23), we have

$$U_n - U_{n-1} = \sum_{j=0}^{1} B_j^*[(1-w_j)/(1-w_j^{\,b})]\, w_j^{\,n-1} \text{ ,n=1,2,…,a-1} \quad (24)$$

The normalization equation (18) can be written as

$$\sum_{n=0}^{a-1} U_n = 1 - \sum_{j=0}^{1} B_j^*/(1-w_j) \quad (25)$$

Solution of the system of (24) and (25) provides

$$U_n = (1/a) - \sum_{j=0}^{1} B_j^* \,[\{(w_j^{\,a} - w_j^{\,b})/a(1-w_j^{\,b})(1-w_j)\} + w_j^{\,n}/(1-w_j^{\,b})]$$

For n= 0,1,…, a-1 (26)

Putting the value of U_n from (26) in (21), we get the expressions for the general time probabilities.

Now, we obtain expressions for the usual performance measures.

1. $L_q \equiv$ mean queue length

$$= \sum_{n=0}^{\infty} nP_n = \sum_{n=0}^{a-1} nU_n + \sum_{n=0}^{\infty} n \sum_{i=0}^{1} P_{(1,0),n,i}$$

$$= (a-1)/2 + \sum_{j=0}^{1} B_j^* \,[\, w_j/(1-w_j)^2 - \{(a-1)/2\}\,(w_j^{\,a} - w_j^{\,b})/(1-w_j)(1-w_j^{\,b}) - w_j\,(1 - w_j^{\,a} - a w_j^{\,a-1}(1-w_j))/(1-w_j)^2(1-w_j^{\,b})] \quad (27)$$

2. $P_{busy} \equiv Pr\{server\ is\ busy\}$

$$= \sum_{n=0}^{\infty} \sum_{i=0}^{1} P_{(1,0),n,i} + \sum_{n_0=a}^{d-1} \sum_{i=0}^{1} P_{(1,1),n_0,i}$$

$$= \sum_{j=0}^{1} B_j^* \{1/(1-w_j)\} + \sum_{n_0=a}^{d-1} \sum_{i=0}^{1} \sum_{j=0}^{1} C_j^* \; g(n_0, i, j) \quad (28)$$

3. $P_{delay} \equiv Pr[an\ arriving\ customer\ has\ to\ wait]$

$= 1 - P_{(0,0),a-1,0} - \sum_{n_0=a}^{d-1} \sum_{i=0}^{1} P_{(1,1),n_0,i}$

$=1 + P_{(0,0),0,0} + \sum_{j=0}^{1} B_j^{*} (1 + w_j^{a-1})/(1 - w_j^{b}) -$

$\frac{2}{a}\{1 - \sum_{j=0}^{1} B_j^{*} (w_j^{a} - w_j^{b})/(1 - w_j^{b})(1 - w_j)\}$ (29)

Waiting-Time Distribution

We consider an arriving customer, whom we call as test customer. Let T_q denote the random variable "the waiting time of a test customer". We define

$W(t) \equiv Pr\ \{0 < T_q \le t\}$

$F_{Tq}(t) \equiv Pr\{ T_q \le t \}$

and our aim is to find closed form expressions for these functions.

We may express W(t) = $W_1(t) + W_2(t) + W_3(t)$ (30)

where

$W_1(t) \equiv Pr\{0 < T_q \le t, (R,0) \equiv (0,0), N \le a-2\}$

$W_2(t) \equiv Pr\{0 < T_q \le t, (R,0) \equiv (1,0), N = nb + m\}$

($a-1 \le m \le b-1$and n=0, 1, 2,…)

$W_3(t) \equiv Pr\{0 < T_q \le t, (R,0) \equiv (1,0), N = nb + m\}$

($0 \le m \le a-2$ and n=0, 1, 2,…)

In order to find theexpressions for above terms, we define

$F_{n,i}(t) \equiv Pr\{0 \le T_q \le t :\ B = i, D = n+1\}$

where D denotes the number of batch departures during the waiting time of the test customer.

In first case the test customer finds the server idle and N= $n \le a-2$ customers in the queue, so he has to wait for a-(n+1) arrivals. Thus his waiting time has an Erlang distribution with parameter a-n-1, and hence

$W_1(t) = \sum_{n=0}^{a-2} (1 - \sum_{l=0}^{a-2-n} e^{-\lambda t} (\lambda t)^{l}/l!)\, P_{(0,0),n,0}$ (31)

In second case, he finds the server busy with non-accessible batch and N = nb +m, $n \ge 0, a-1 \le m \le b-1$ customers in the queue , so his waiting time is not affected by the number of arrivals after his joining the queue and consists of the interval taken up to the (n+1)th subsequent batch departures. Thus, we have

$W_2(t) = \sum_{n=0}^{\infty} \sum_{m=a-1}^{b-1} \sum_{i=0}^{1} P_{(1,0),nb+m,i}\, F_{n,i}(t)$ (32)

In third case he finds the server busy with non-accessible batch and N = nb + m, $n \ge 0$, $0 \le m \le a-2$ customers in the queue, then, he has to wait for the maximum time taken by either n+1 departures or

a-1-m arrivals. So, we have

$W_3(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{a-2} \sum_{i=0}^{1} (P_{(1,0),nb+m,i}\, F_{n,i}(t)\,)\,(1- \sum_{l=0}^{a-2-m} e^{-\lambda t}\, (\lambda t)^{l}/l!)$ (33)

In the above expression, the only unknown quantities are

$A_j(t) = \sum_{i=0}^{1} f(i, w_j) \sum_{n=0}^{\infty} w_j^{nb} F_{n,i}(t)$ j=0,1(35)

But we have the quantities $A_j(t)$ to be

$$A_j(t) = \{B_j^*/C_j^*(1 - w_j^b)\}(1 - e^{-\lambda((1-w_j)/w_j)t}) \quad (36)$$

Using (36) in (34), we obtain

$$W(t) = P_{delay} + (1 - P_{delay} - U_{a-1})\, C_0(t) - \sum_{n=0}^{a-2} U_n C_n(t) - \sum_{j=0}^{1} B_j^*\,(1/(1-w_j))e^{-\lambda((1-w_j)/w_j)t} - \sum_{j=0}^{1} B_j^*\{1/(1-w_j^b)\}\sum_{n=0}^{a-2} w_j^n\, C_n(t)\,(1 - e^{-\lambda((1-w_j)/w_j)t}) \quad (37)$$

where

$C_n(t) = \sum_{l=0}^{a-2-n} e^{-\lambda t}(\lambda t)^l/l!)$

From (37), it is evident that

$\lim_{t\to\infty} W(t) = P_{delay} = Pr\{T_q > 0\}$

Now, $F_{Tq}(t)$ is given by

$$F_{Tq}(t) = [1 - Pr\{T_q > 0\}] + W(t) \quad (38)$$

The rth moment of T_q *can be* calculated through

$E[T_q^r] = r\int_0^{\infty} t^{r-1}(1 - F_{Tq}(t))\, dt$

In particular, the first moment of T_q is found to be

$$E[T_q] = (1/\lambda)\,[(a/2)\{1 + \sum_{j=0}^{1} B_j^*(w_j^a + w_j^b)/(1-w_j)(1-w_j^b)\} - \tfrac{1}{2}\{1 - \sum_{j=0}^{1} B_j^*(1+w_j)(w_j^a - w_j^b)/(1-w_j)^2(1-w_j^b)\} - (a-1)\sum_{n_0=a}^{d-1}\sum_{i=0}^{1}\sum_{j=0}^{1} C_j^*\, g(n_0, i, j)] \quad (39)$$

CONCLUSION

Queuing performance measures are very important for maintenance and improvement of the queuing system. Queue length and waiting time are a matter of concern for the customer as well as the organizer of queue. Information about the busy period of server helps to know about the need ofintroducing new server. For the proposed model steady-state queue length distribution and waiting time distributions are derived. Probabilities for the server to be busy and an arriving customer has to wait are also found. As the service time distribution is Coxian-2 type, many real- life queues may be modelled by this queuing system.

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